Electro-aerodynamic instability of a thin dielectric liquid sheet sprayed with an air stream

M. F. El-Sayed*

Department of Mathematics and Computer Science, Faculty of Science, United Arab Emirates University, P.O. Box 17551, Al Ain, United Arab Emirates

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The instability of a thin sheet of dielectric liquid moving in the same direction as an air stream in the presence of a uniform horizontal electric field is studied theoretically. It is found that aerodynamic-enhanced instability occurs if the Weber number is much less than a critical value related to the ratio of the air and liquid stream velocities, the electric field, and the dielectric constant values. The electric field is found to have a stabilizing effect, and there exists a critical Weber number above which instability is suppressed by the surface tension effect. The condition for disintegrating the sheet is obtained in terms of the electric field values, and some limiting cases are recovered. [S1063-651X(99)01312-4]

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I. INTRODUCTION

The instability of a thin liquid sheet is of considerable scientific and technological importance, and has been extensively studied in the past. Practical applications of plane liquid sheets can be found in power generation and propulsion systems [1], chemical and pharmaceutical processes [2], surface curtain coatings, and in the adhesive industry [3]. In connection with atomization, the stability and breakup process of thin liquid sheets has been investigated by Squire [4], Hagerty and Shea [5], Dombrowski and Johns [6], among others. Most of the past studies in this regard have been summarized by Lefebvre [1]. For recent excellent investigations about the subject and to different physical problems of interest, see Refs. [7–13]. In all the above-mentioned studies, the effect of an applied electric field, which has numerous applications in astrophysics, chemical engineering, and industry [14], has not been taken into consideration. If the thin liquid sheet is stressed by an applied electric field, then the conditions of stability will be considerably modified.

The fundamental principle of the disintegration of a liquid sheet into drops consists of the following mean stages [15]: (i) initiation of small disturbances at the surface of the liquid, (ii) the growth of these disturbances until ligaments or threads are formed, (iii) the breakup of ligaments or threads into drops, and (iv) further breakup of these drops in their movement through air. Four modes of disintegration have been defined [16]: rim, wavy sheet, perforated sheet, and air impact. *Rim* disintegration occurs due to the contraction of the sheet edges under the effect of surface tension, whereas threads are pulled out of the rim during contraction. In *wavy sheet* disintegration, any small protuberance on the sheet is subjected to two opposing forces: surface tension force which draws the liquid back to the original undisturbed shape, and aerodynamic force which pulls the liquid outward. If the aerodynamic force exceeds the surface tension force, then any small disturbance present in the sheet will grow rapidly, causing sheet instability. In *perforated sheet* disintegration, disturbances on the sheet puncture it when the sheet becomes thin enough, and the resulting holes expand regularly by surface tension until they coalesce, forming threads. In *air impact* disintegration, the disruption of the liquid is very near to that of a twin-fluid nozzle, where two streams of air and liquid are caused to impinge together.

Squire [4] and Hagerty and Shea [5] examined the instability of an inviscid liquid sheet in a stationary gaseous medium. Their analytical results show that the surface tension forces always tend to stabilize any proturberances, and when the aerodynamic forces resulting from the interaction between the liquid sheet and the ambient gas are dominant, the disturbances will be further enhanced, i.e., the sheet will become unstable and evantually disintegrate. Dombrowski and Johns [6] extended the analysis by including the effect of liquid viscosity, and their results are only valid for very large liquid velocities (or Weber numbers) due to the approximations made there. The hydrodynamic of capillary waves on free, viscous, liquid sheets has been investigated by Joosten *et al.* [17]. The effect of the forces responsible for the stability of the sheet has been accounted for in the boundary conditions. Their experimental results, obtained by laser beat spectroscopy, were in qualitative agreement with theory. The weakly nonlinear surface waves on liquid sheets in which long-range interaction forces are operative is reported by Joosten [17].

The sheet oscillations can be divided into symmetrical and antisymmetrical modes. For the former, the displacement of the corresponding points on the free surface is equal in magnitude and in the opposite direction, and for the latter the displacements are in the same direction. Examining the aerodynamic instability of liquid sheets moving in still air, Squire [4] found that the degree of instability of symmetrical oscillations is very much less than that of the antisymmetrical ones. It might be mentioned that practically any finite width liquid sheet at low velocity would be destroyed by the edge of the surface tension effects [18]. Also for liquid sheets produced by swirl nozzles, the liquid sheet would become a liquid bubble at sufficiently low velocity [1].

The present investigation, therefore, deals with the sheet disintegration through aerodynamic instability when the di-

*Also at Department of Mathematics, Faculty of Education, Ain Shams University, Roxy, Cairo, Egypt. Electronic address: elsayed@nyx.uaeu.ac.ae

electric sheet is moving in the same direction as an air stream, and in the presence of a uniform horizontal electric field. This problem, to the best of my knowledge, has not been investigated yet. The investigation given below treats the sheet as two dimensional, irrotational, and of constant thickness, considering only antisymmetric oscillations (predominant for instability as mentioned by Squire [4]) of the moving sheet.

II. BASIC AND PERTURBATION EQUATIONS

Consider a two-dimensional irrotational sheet of dielectric liquid of density ρ_1, dielectric constant ε_1, surface tension T, and thickness $h=2a$, moving with velocity U_1 along an air stream (on both sides of the sheet), which is of density ρ_2, dielectric constant ε_2, and moving with velocity U_2. The whole system is influenced by a uniform horizontal electric field E_0 in the positive x direction, where the origin is located at the midplane of the liquid sheet. We also assume that the quasistatic approximation is valid here; then the electric field can be derived from a scalar potential Ψ (i.e., $\mathbf{E}=-\nabla\Psi$). Accordingly, Maxwell's equations reduce to

$$\nabla\times\mathbf{E}=\mathbf{0}, \tag{1}$$

$$\nabla\cdot(\varepsilon\mathbf{E})=\mathbf{0}. \tag{2}$$

In aerial application of spray, when nozzles are directed backward, we have the case of a liquid sheet moving with an air stream. Consider this sheet to be slightly disturbed; we will then show that the forces acting on the disturbed interface will be the surface tension and the applied electric field, which tend to draw the liquid back to its original undisturbed shape, while the surrounding air velocity will be increased, creating a local decrease in pressure which tends to pull the liquid outward.

Let Φ_j' and Ψ_j', $j=1,2$, be the disturbance potentials of the motion, and the electric field in liquid and air, respectively, then the velocity potential and the total electric field in the liquid and air can be expressed in the form

$$\phi_j=-U_jx+\Phi_j', \quad j=1,2, \tag{3}$$

$$\mathbf{E}_j=E_0\mathbf{i}-\nabla\Psi_j', \quad j=1,2, \tag{4}$$

where the velocity and electric potentials Φ_j' and Ψ_j', $j=1,2$, satisfy Laplace's equations

$$\nabla^2\Phi_j'=\mathbf{0}, \tag{5}$$

$$\nabla^2\Psi_j'=\mathbf{0}. \tag{6}$$

The displacement of sheet surface $\eta=\eta(x,t)$ can be expressed in the form $\eta=A\exp\{in(x-ct)\}$, where A is the amplitude of the wave, $n(=2\pi/\lambda)$ is the wave number, λ being the wavelength of oscillations, c is the wave propagation velocity, and t is the time. Now, if A is expressed in the form $A=A_0\exp(\beta t)$, then instability will exist only if β is found to be real and positive, and η can now be written in the form

$$\eta=A_0\exp\{i(nx-\sigma t)\}, \tag{7}$$

where $\sigma=nc+i\beta$. Note that, in this case, unstable waves will result if the aerodynamic forces exceed the surface tension and electric field forces.

III. BOUNDARY CONDITIONS AND SOLUTIONS

Since the equation of the upper surface is given by $y=\eta+a$, where $\eta=\eta(x,t)$, then

$$\frac{d\eta}{dt}=\frac{\partial\eta}{\partial t}+\frac{\partial\eta}{\partial x}\frac{dx}{dt}.$$

Neglecting second-order terms, then the kinematic boundary conditions can be written in the form

$$\frac{\partial\eta}{\partial t}+U_1\frac{\partial\eta}{\partial x}=-\frac{\partial\Phi_1'}{\partial y} \quad \text{at} \quad y=a, \tag{8}$$

$$\frac{\partial\eta}{\partial t}+U_2\frac{\partial\eta}{\partial x}=-\frac{\partial\Phi_2'}{\partial y} \quad \text{at} \quad y=a, \tag{9}$$

where Φ_2' refers to the upper region $y>a$.

Suitable expressions for Φ_1',Φ_2', which satisfy the above conditions (8) and (9) together with Laplace's equations (5), are found by inspection to be

$$\Phi_1'=i\eta\left(\frac{\sigma}{n}-U_1\right)\frac{\sinh ny}{\cosh na}, \tag{10}$$

$$\Phi_2'=-i\eta\left(\frac{\sigma}{n}-U_2\right)\exp\{-n(y-a)\}. \tag{11}$$

The applied electric field should satisfy the following boundary conditions [19–21].

(i) The tangential component of the electric field is continuous at the interface, i.e.,

$$\frac{\partial\Psi_1'}{\partial x}=\frac{\partial\Psi_2'}{\partial x} \quad \text{at} \quad y=a. \tag{12}$$

(ii) The normal component of the electric displacement is continuous at the interface, i.e.,

$$\mathbf{N}\cdot\varepsilon_1\mathbf{E}_1=\mathbf{N}\cdot\varepsilon_2\mathbf{E}_2 \quad \text{at} \quad y=a, \tag{13}$$

where the unit normal vector $\mathbf{N}$ to the interface $F=y-\eta(x,t)=0$, to the first-order terms, is given by

$$\mathbf{N}=\nabla F/|\nabla F|=-in\eta\mathbf{i}+\mathbf{j}, \tag{14}$$

where $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors in the x and y directions, respectively.

Using Eqs. (4), (7), and (14), the suitable expressions for Ψ_1',Ψ_2', which satisfy the above conditions (12) and (13), together with Laplace's equations (6), can be written in the form

$$\Psi_1'=\frac{iE_0(\varepsilon_2-\varepsilon_1)\eta}{(\varepsilon_2+\varepsilon_1\coth na)}\frac{\sinh ny}{\sinh na}, \tag{15}$$

$$\Psi_2'=\frac{iE_0(\varepsilon_2-\varepsilon_1)\eta}{(\varepsilon_2+\varepsilon_1\coth na)}\exp\{-n(y-a)\}. \tag{16}$$

The pressure condition at the free surface is [20]

$$P_1-P_2+E_0\left[\varepsilon_2\frac{\partial\Psi_2'}{\partial x}-\varepsilon_1\frac{\partial\Psi_1'}{\partial x}\right]=-T\frac{\partial^2\eta}{\partial x^2}\quad\text{at}\quad y=a, \tag{17}$$

where T is the surface tension coefficient.

IV. DISINTEGRATION OF THE LIQUID SHEET

Now applying the pressure equation of incompressible fluid for both liquid and air, i.e.,

$$\frac{P}{\rho}+\frac{V^2}{2}-\frac{\partial\Phi'}{\partial t}=\text{const},$$

in which the gravitational force is neglected, then the pressure condition (17) reduces to

$$\rho_1\left[\frac{\partial\Phi_1'}{\partial t}+U_1\frac{\partial\Phi_1'}{\partial x}\right]-\rho_2\left[\frac{\partial\Phi_2'}{\partial t}+U_2\frac{\partial\Phi_2'}{\partial x}\right]+E_0\left[\varepsilon_2\frac{\partial\Psi_2'}{\partial x}-\varepsilon_1\frac{\partial\Psi_1'}{\partial x}\right]=-T\frac{\partial^2\eta}{\partial x^2}\quad\text{at}\quad y=a, \tag{18}$$

which after substituting for Φ_j' and Ψ_j', $j=1,2$, from Eqs. (10), (11), (15), and (16) yields the following dispersion relation:

$$\rho_1\left(\frac{\sigma}{n}-U_1\right)^2\tanh na+\rho_2\left(\frac{\sigma}{n}-U_2\right)^2-\frac{E_0^2(\varepsilon_2-\varepsilon_1)^2}{(\varepsilon_2+\varepsilon_1\coth na)}=Tn. \tag{19}$$

Dividing Eq. (19) by $\rho_1U_1^2$, and calling $\rho_2/\rho_1=r$ and $U_2/U_1=U$, we get

$$\left(\frac{\sigma}{nU_1}-1\right)^2+r\left(\frac{\sigma}{nU_1}-U\right)^2\coth na=\frac{\coth na}{U_1^2}\left[\frac{E_0^2(\varepsilon_2-\varepsilon_1)^2}{(\varepsilon_2+\varepsilon_1\coth na)}+Tn\right]. \tag{20}$$

Solving Eq. (20) for σ/nU_1, we obtain

$$\frac{\sigma}{nU_1}=\frac{1+rU\coth na}{1+r\coth na}\pm\frac{\sqrt{\coth na}}{1+r\coth na}\{-r(1-U)^2+(1/\rho_1U_1^2)(1+r\coth na)[E_0^2(\varepsilon_2-\varepsilon_1)^2\times(\varepsilon_2+\varepsilon_1\coth na)^{-1}+Tn]\}^{1/2}. \tag{21}$$

Unstable waves will occur only if β is real and positive, i.e., $\sigma(=nc+i\beta)$ is to be complex. Therefore, Eq. (21) yields

$$\frac{c}{U_1}=\frac{1+rU\coth na}{1+r\coth na} \tag{22}$$

and

$$\frac{\beta}{U_1}=\frac{n\sqrt{\coth na}}{1+r\coth na}\{r(1-U)^2-(1+r\coth na)[(E_0^2/\rho_1U_1^2)\times(\varepsilon_2-\varepsilon_1)^2(\varepsilon_2+\varepsilon_1\coth na)^{-1}+naW]\}^{1/2}, \tag{23}$$

where $W\ (=T/\rho_1U_1^2a)$ is the Weber number. Since in practical applications na is very small [4], then considering the approximation $\tanh na\simeq na$ in Eq. (23), we obtain

$$\frac{\beta a}{U_1}=\frac{\sqrt{rna}(1+r/na)^{-1}}{r\sqrt{\varepsilon_2 na+\varepsilon_1}}\left\{r(1-U)^2-(na+r)\times\left[\frac{E_0^2(\varepsilon_2-\varepsilon_1)^2}{\rho_1U_1^2(\varepsilon_2na+\varepsilon_1)}+W\right]\right\}^{1/2}. \tag{24}$$

The minimum wavelength for instability to occur is found to be with the condition $\beta=0$, for which it can be written that

$$na=-\tfrac{1}{2}\{(E_0^2/\rho_1U_1^2W\varepsilon_2)(\varepsilon_2-\varepsilon_1)^2-r[(1-U)^2W^{-1}-1]\}+\tfrac{1}{2}(\{(E_0^2/\rho_1U_1^2W\varepsilon_2)(\varepsilon_2-\varepsilon_1)^2-r[(1-U)^2W^{-1}-1]\}^2-4r\{(E_0^2/\rho_1U_1^2W\varepsilon_2)(\varepsilon_2-\varepsilon_1)^2-(\varepsilon_1/\varepsilon_2)[(1-U)^2W^{-1}-1]\})^{1/2}. \tag{25}$$

Then the corresponding wavelength $\lambda_{\min}$ will be

$$\lambda_{\min}=2\pi/n, \tag{26}$$

where n is given by Eq. (25).

From Eqs. (25) and (26), it can be concluded that the condition for $\lambda_{\min}$ to be infinite is

$$\frac{\varepsilon_1}{\varepsilon_2}\left\{\frac{(1-U)^2}{W}-1\right\}=\frac{E_0^2(\varepsilon_2-\varepsilon_1)^2}{\rho_1U_1^2W\varepsilon_2}. \tag{27}$$

Hence, for instability it must be

$$\left\{\frac{(1-U)^2}{W}-1\right\}>\frac{E_0^2(\varepsilon_2-\varepsilon_1)^2}{\rho_1U_1^2W\varepsilon_1}, \tag{28}$$

which is the condition for disintegration of the liquid sheet in the presence of the applied electric field. Therefore, the liquid sheet in this case will disintegrate according to the wavy sheet mode [16]. Equation (28) reduces to the same condition obtained earlier by Rashed *et al.* [22], in the absence of the electric field (i.e., when $E_0=0$). It reduces also to the same equation obtained by Squire [4] in the limit when both E_0 and U vanish. Note that if the condition given by Eq. (28) does not exist, then the sheet will not disintegrate according to the wavy sheet mode, and the disintegration will occur according to another mode, which is referred to as the perforated sheet mode [22]. Also, the wavelength for maximum instability can be obtained when β is maximum, i.e., if the condition $d\beta/dn=0$ [using Eq. (24)] is satisfied. The resulting formula, following Squire [4] and Rashed *et al.* [22], can be obtained, and this will not be given here because it is very lengthy. For an inviscid liquid sheet, it is found that only one mode of instability coexists, namely, aerodynamic-enhanced, and the mechanism of instability is due to the interfacial pressure fluctuations. In this case, the aerodynamic forces resulting from the interaction between the thin liquid sheet and the ambient air are found to be responsible for the instability of the inviscid sheet. In the limit of minimum and maximum wavelengths, it is found also that instability is due to the velocity jump across the two liquid-air interfaces, and from one air stream to the other, hence it is related to the classical Kelvin-Helmholtz instability.

Finally, the following conclusions, on the analysis of electro-aerodynamic instability of liquid sheets sprayed with an air stream, can be outlined.

(i) Instability occurs if the Weber number W is much less than $(1-U)^2$ in accordance with Eq. (28), and in this case the surface tension, affected through the Weber number, always acts as a stabilizing agent.

(ii) The presence of the electric field reduces the range of instability rather than in the absence of it, therefore the applied electric field has a stabilizing effect on the aerodynamic instability of the liquid sheet.

(iii) In the limiting case when $U_2=0$, i.e., when the liquid sheet moves with velocity U_1 in motionless air, the values of the Weber number W in this case should be very much less than those values (obtained previously by Squire [4]) if the electric field is absent.

(iv) There exists a critical Weber number below which the surface tension is the source of instability, whereas above this critical value, instability is suppressed by the surface tension effect, and is promonted by the aerodynamic interaction between the liquid and air phase in the presence of the applied electric field.

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